Topic 4

Graphs and transformations

Bronze, Silver, Gold
Worksheets for
AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

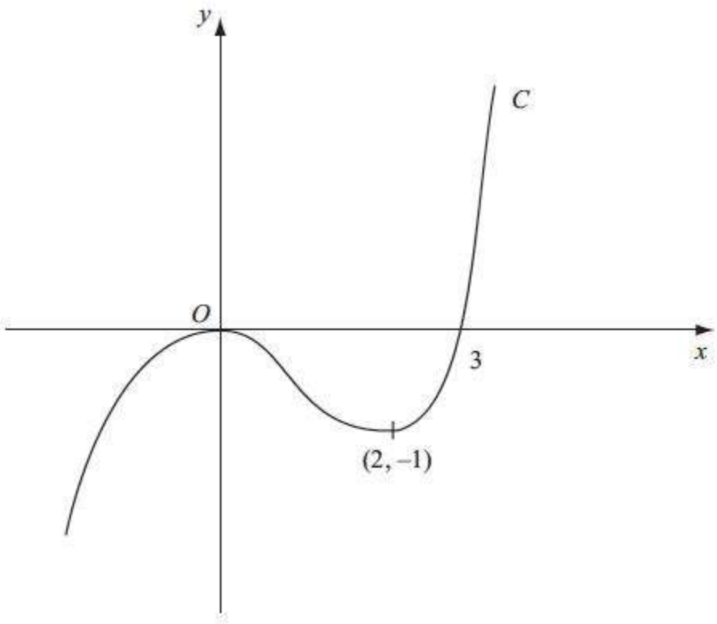


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$. There is a maximum at (0, 0), a minimum at (2, −1) and C passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x + 3)$,

(3)

(b) $y = \mathrm{f}(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

(Total for Question 1 is 6 marks)

Q2

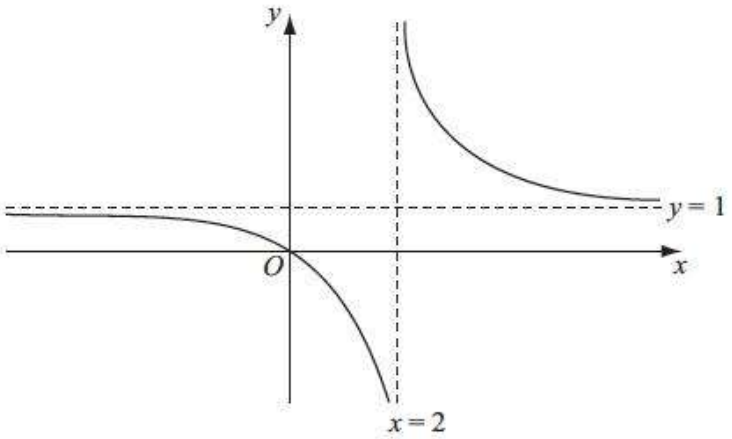


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x)\frac{x}{x-2}, \quad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) Sketch the curve with equation $y = \mathrm{f}(x - 1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x - 1)$ crosses the coordinate axes.

(4)

(Total for Question 2 is 7 marks)

Q3

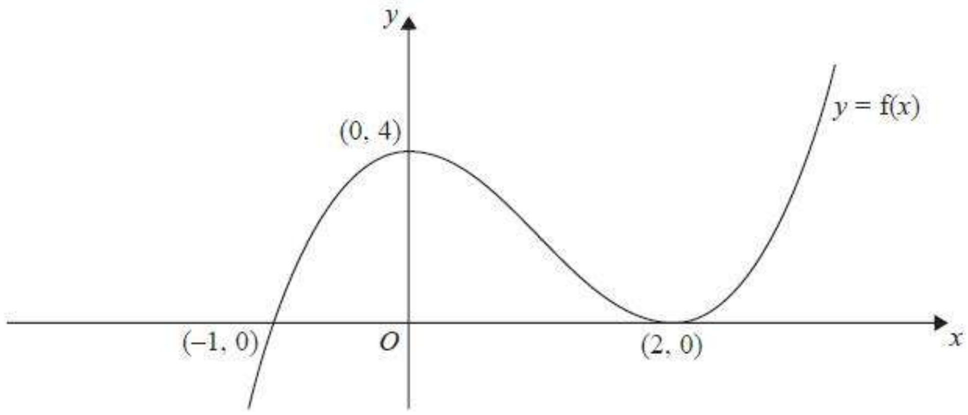


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$

The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$

The curve C has a maximum at the point $(0, 4)$

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a, b and c are integers.

Calculate the values of a, b and c.

(5)

(b) Sketch the curve with equation $y = \mathrm{f}\left(\frac{1}{2}\right)$

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

(3)

(Total for Question 3 is 8 marks)

Q4

(a) Sketch the graphs of

$$y = x(x + 2)(3 - x)$$

$$y = -\frac{2}{x}$$

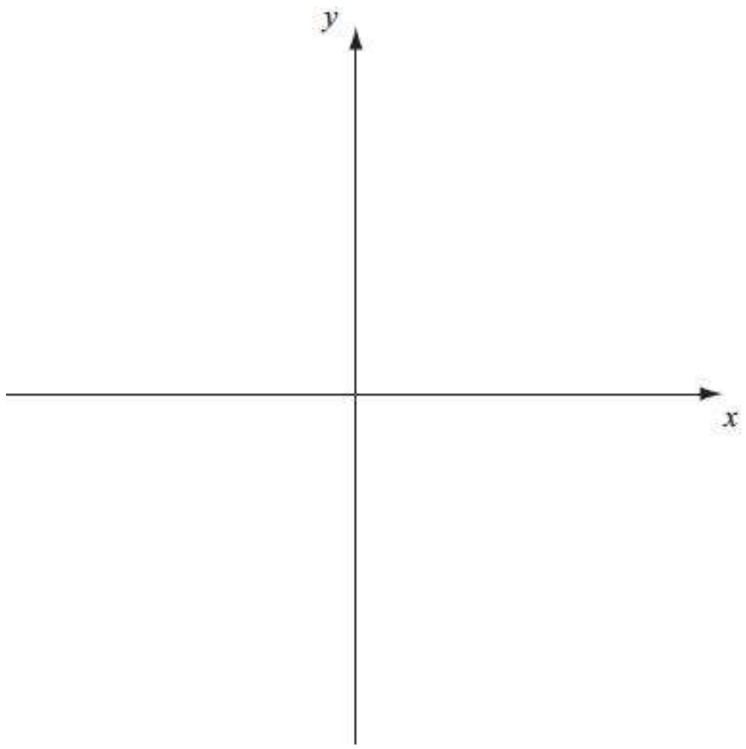
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x)+\frac{2}{x}=0$$

(2)

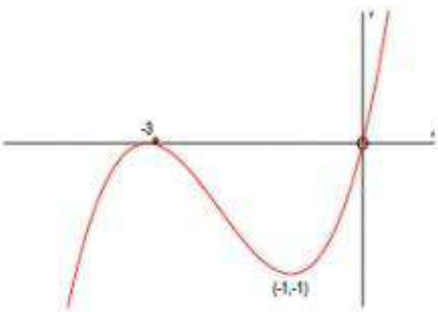
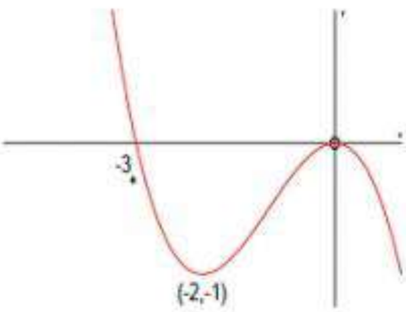


(Total for Question 4 is 8 marks)

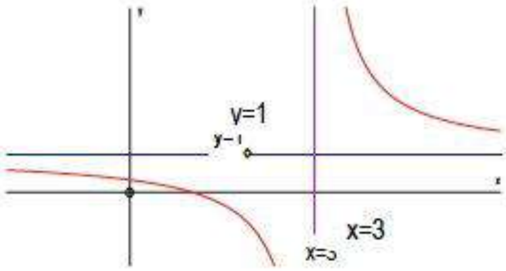
End of Questions

Bronze Mark Scheme

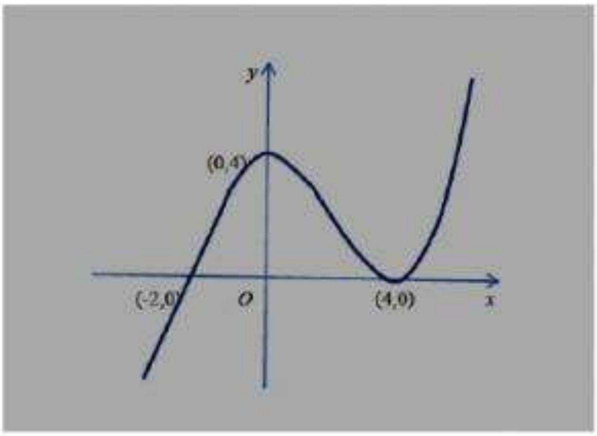
Q1

Question Number	Scheme		Marks	
(a)		Shape, touching the x-axis at its maximum. Through $(0,0)$ & -3 marked on x-axis, or $(-3,0)$ seen. Allow $(0,-3)$ if marked on the x-axis. Marked in the correct place, but 3, is A0. Min at $(-1,-1)$	M1 A1 A1	 (3)
(b)		Correct shape (top left - bottom right) Through -3 and max at $(0, 0)$. Marked in the correct place, but 3, is B0. Min at $(-2,-1)$	B1 B1 B1	 (3) **[6]**
(a)	M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 1st A1 for curve passing through -3 and the origin. Max at $(-3,0)$ 2nd A1 for minimum at $(-1,-1)$. Can simply be indicated on sketch.			
(b)	1st B1 for the correct shape. A negative cubic passing from top left to bottom right. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for curve passing through $(-3,0)$ having a max at $(0, 0)$ and no other max. 3rd B1 for minimum at $(-2,-1)$ and no other minimum. If in correct quadrant but labelled, e.g. $(-2,1)$, this is B0. In each part the $(0, 0)$ does not need to be written to score the second mark… having the curve pass through the origin is sufficient. The last mark (for the minimum) in each part is dependent on a sketch being attempted, and the sketch must show the minimum in approximately the correct place (not, for example, $(-2,-1)$ marked in the wrong quadrant). The mark for the minimum is not given for the coordinates just marked on the axes unless these are clearly linked to the minimum by vertical and horizontal lines.			

Q2

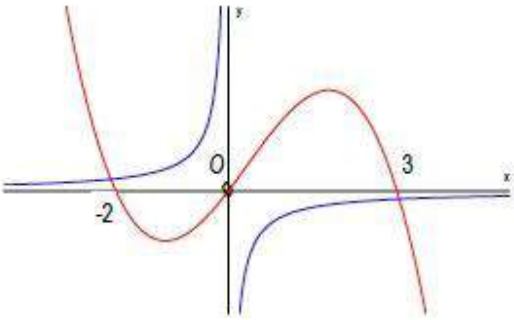
Question Number	Scheme	Marks
(a)	 Correct shape with a single crossing of each axis	B1
	$y = 1$ labelled or stated	B1
	$x = 3$ labelled or stated	B1 (3)
(b)	Horizontal translation so crosses the x-axis at (1, 0)	B1
	New equation is $(y =)\dfrac{x \pm 1}{(x \pm 1) - 2}$	M1
	When $x = 0$ $\quad y =$	M1
	$= \dfrac{1}{3}$	A1
		(4) **7**
	Notes	
(b)	B1 for point (1,0) identified - this may be marked on the sketch as 1 on x axis. Accept $x = 1$. 1st M1 for attempt at new equation and either numerator or denominator correct 2nd M1 for setting $x = 0$ in their new equation and solving as far as $y = \ldots$ A1 for $\frac{1}{3}$ or exact equivalent. Must see $y = \frac{1}{3}$ or $(0, \frac{1}{3})$ or point marked on y-axis. **Alternative** $\text{f}(-1) = \dfrac{-1}{-1-2} = \dfrac{1}{3}$ scores M1M1A0 unless $x = 0$ is seen or they write the point as $(0, \frac{1}{3})$ or give $y = 1/3$ Answers only: $x = 1$, $y = 1/3$ is full marks as is (1,0) (0, 1/3) Just 1 and 1/3 is B0 M1 M1 A0 Special case : Translates 1 unit to left (a) B0, B1, B0 (b) Mark (b) as before May score B0 M1 M1 A0 so 3/7 or may ignore sketch and start again scoring full marks for this part.	

Q3

Question Number	Scheme	Notes	Marks
(a)	$f(x)=(x+1)(x-2)^2$	M1: Either stating or writing down that $(x\pm1)$ **or** $(x\pm2)$ is a factor – may be implied by their f(x)	M1A1B1
		A1: **Both** $(x+1)$ **and** $(x-2)$ are factors - may be implied by their f(x)	
		B1: y or f(x)= $(x+1)(x-2)^2$	
	$=(x+1)(x^2-4x+4)=x^3-3x^2+4$	M1: Multiplying out a quadratic to get 3 terms and then multiplying by the linear term to form a cubic.	M1A1
		A1: x^3-3x^2+4 or $a=-3$, $b=0$, $c=4$	
			(5)
(b)		Same shape and position (ignore any coordinates) with the maximum on the y-axis	B1
		y intercept = 4 or their 'c'	B1ft
		x coordinates at -2 and 4 or marked as coordinates. Allow (0, -2) and (0, 4) if they are marked in the correct position. The curve must cross or at least stop at x = -2	B1
			(3)
			[8]
(a) Way 2	$x=0, y=4\Rightarrow c=4$	Uses (0, 4) to obtain c = 4 (can be just stated)	B1
	$x=-1, y=0\Rightarrow -1+a-b+c=0$ $x=2, y=0\Rightarrow 8+4a+2b+c=0$	Uses both (-1, 0) and (2, 0) in $y=x^3+ax^2+bx+c$ to form 2 simultaneous equations. Allow the equations to contain c here.	M1
	$a-b=-3$ $4a+2b=-12$ $\Rightarrow a=..\ or\ b=...$	Solves simultaneously with a value for c to obtain a value for a or a value for b	M1
	Either a = -3 **or** b = 0		A1
	Both a = -3 **and** b = 0		A1

(a) **Way 3**	$\frac{dy}{dx} = 3x^2 + 2ax + b$	M1: $x^n \rightarrow x^{n-1}$ at least once including c → 0	M1
	$x = 0 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow b = 0$	Correct value for b	A1
	$x = 0, y = 4 \Rightarrow c = 4$	Uses (0, 4) to obtain $c = 4$ (can be just stated)	B1
	$3(2)^2 + 2a(2) + b = 0$ or $(-1)^3 + a(-1)^2 + b(-1) + 4 = 0$	Obtains an equation in a	M1
	$a = -3$	Correct value for a	A1
			(5)
	Special case: A common incorrect approach is to assume the cubic is of the form e.g. $f(x) = x(x \pm 1)(x \pm 2) + 4$ This scores B1 only for $c = 4$		
			[8]

Q4

Question Number	Scheme	Marks
(a)	 (i) correct shape (-ve cubic) Crossing at (-2, 0) Through the origin Crossing at (3,0) (ii) 2 branches in correct quadrants not crossing axes One intersection with cubic on each branch	B1 B1 B1 B1 B1 B1 (6)
(b)	"2" solutions Since only "2" intersections	B1ft dB1ft (2) **8**
	Notes	
(b)	B1ft for a value that is compatible with their sketch dB1ft This mark is dependent on the value being compatible with their sketch. For a comment relating the number of solutions to the number of intersections. [Only allow 0, 2 or 4]	

Silver Questions

Calculators may not be used

The total mark for this section is 30

Q1

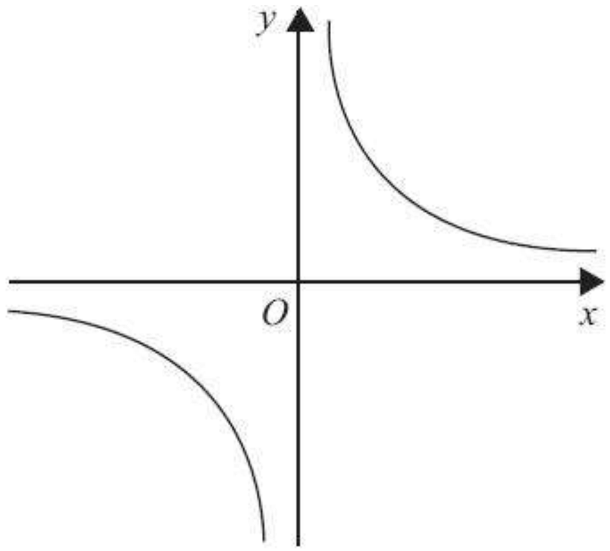


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$

(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \neq -2$,

showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)

(Total for Question 1 is 5 marks)

Q2

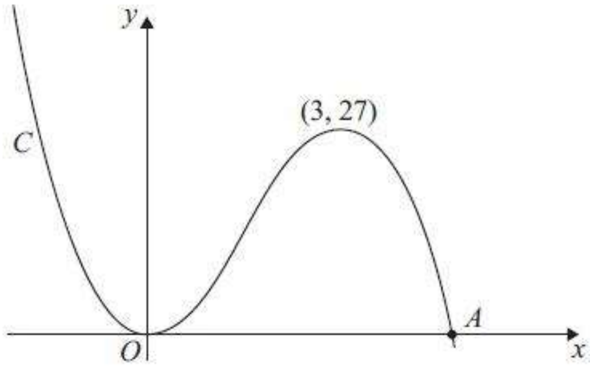


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = f(x+3)$

(ii) $y = f(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

(Total for Question 2 is 8 marks)

Q3

(a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

(Total for Question 3 is 9 marks)

Q4

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

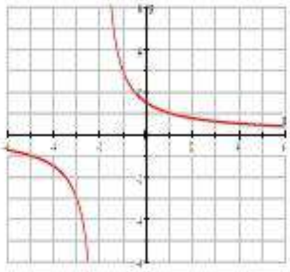
(c) Hence find the exact values of k for which l is a tangent to C.

(3)

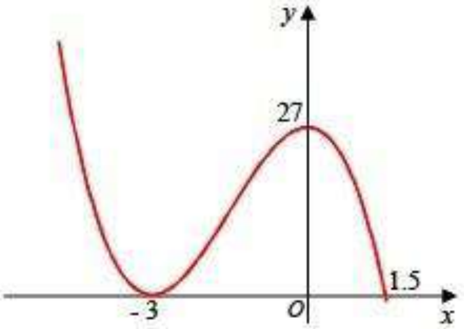
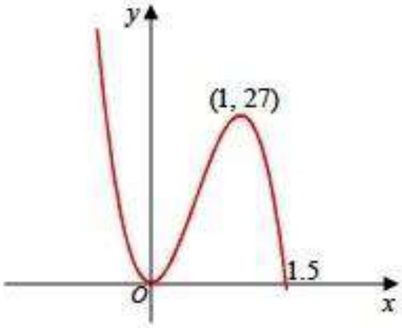
(Total for Question 4 is 8 marks)

Silver Mark Scheme

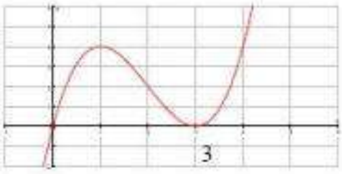
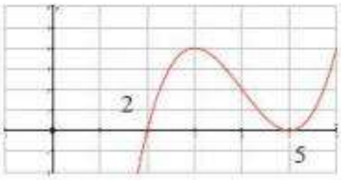
Q1

Question number	Scheme	Marks
	(a) 	
	Translation parallel to x-axis	M1
	Top branch intersects +ve y-axis Lower branch has no intersections No obvious overlap	A1
	$\left(0, \frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y- axis	B1 (3)
	(b) $x = -2, \quad y = 0$	B1, B1 (2)
S.C.	[Allow ft on first B1 for $x = 2$ when translated "the wrong way" but must be compatible with their sketch.]	
		5
(a)	M1 for a horizontal translation – two branches with one branch cutting y – axis only. If one of the branches cuts both axes (translation up and across) this is M0. A1 for a horizontal translation to left. Ignore any figures on axes for this mark. B1 for correct intersection on positive y-axis. More than 1 intersection is B0. x=0 and y = 1.5 in a table alone is insufficient unless intersection of their sketch is with +ve y-axis. A point marked on the graph overrides a point given elsewhere.	
(b)	1st B1 for $x = -2$. NB $x \neq -2$ is B0. Can accept $x = +2$ if this is compatible with their sketch. Usually they will have M1A0 in part (a) (and usually B0 too) 2nd B1 for $y = 0$.	
S.C.	If x = -2 and y =0 and some other asymptotes are also given award B1B0 The asymptote equations should be clearly stated in part (b). Simply marking x =–2 or y =0 on the sketch is insufficient <u>unless</u> they are clearly marked "asymptote x = -2" etc.	

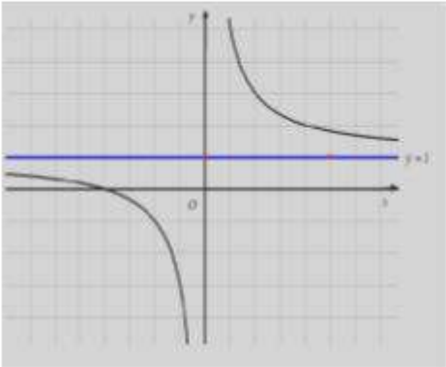
Q2

Question Number	Scheme	Marks
(a)	{Coordinates of A are} $(4.5, 0)$ See notes below	B1 [1]
(b)(i)	 Horizontal translation -3 and their ft 1.5 on postitive x-axis Maximum at 27 marked on the y-axis	M1 A1 ft B1 [3]
(ii)	 Correct shape, minimum at $(0, 0)$ and a maximum within the first quadrant. 1.5 on x-axis Maximum at $(1, 27)$	M1 A1 ft B1 [3]
(c)	$\{k =\}\ -17$	B1 [1] 8
	Notes	
(a)	**B1:** For stating either $x = 4.5$ or $\frac{9}{2}$ or $\frac{18}{4}$ etc. or $A = 4.5$ or $\frac{9}{2}$ or $(4.5, 0)$. Can be written on graph. Allow $(0, 4.5)$ marked on curve for B1. Otherwise $(0, 4.5)$ without reference to any of the above is B0.	
(b)(i)	**M1:** for any horizontal (left-right) translation where minimum is still on x-axis not at $(0, 0)$. Ignore any values. **A1ft:** for -3 **(NOT 3)** and 1.5 (or their x in part (a) $- 3$) *evaluated* and marked on the positive x-axis. Allow $(0, -3)$ and/or $(0, \text{ft } 1.5)$ rather than $(-3, 0)$ and $(\text{ft } 1.5, 0)$ if marked in the "correct" place on the x-axis. **Note:** Candidate *cannot* gain this mark if their x in part (a) is less than 3. **B1:** Maximum at 27 marked on the y-axis. **Note**: the maximum must be on the y-axis for this mark.	
(ii)	**M1:** for correct shape with minimum still at $(0, 0)$ and a maximum within the first quadrant. Ignore values. **A1ft:** for $\dfrac{\text{their } x \text{ in part } (a)}{3}$; as intercept on x-axis eg: $\dfrac{4.5}{3}$ or 1.5 or $\dfrac{3}{2}$ or $\dfrac{9}{6}$ **Note:** a generalised $\dfrac{A}{3}$ is A0. Allow $(0, \text{ft } 1.5)$ rather than $(\text{ft } 1.5, 0)$ if marked in the "correct" place on the x-axis. **B1:** Maximum at $(1, 27)$ or allow 1 marked on the x-axis and the corresponding 27 marked on the y-axis. **Note: Be careful to look at the correct graph. The candidate may draw another graph to help them to answer part (c).** **Note:** You can recover (b)(i) $(-3, 0)$ and $(\text{ft } 1.5, 0)$ or in (b)(ii) $(\text{ft } 1.5, 0)$ as *correct coordinates only* in candidate's working if these are not marked on their sketch(es).	
(c)	**B1:** for $(k =)\ -17$ only. **BEWARE**: This could be written in the middle or at the bottom of a page.	

Q3

Question Number	Scheme	Marks
Q (a)	$x(x^2-6x+9)$	B1
	$= x(x-3)(x-3)$	M1 A1 (3)
(b)	 Shape 	B1
	<u>Through</u> origin (<u>not</u> touching)	B1
	Touching x-axis only once	B1
	Touching at (3, 0), or 3 on x-axis	B1ft (4)
	[Must be on graph not in a table]	
(c)	 Moved horizontally (either way)	M1
	(2, 0) and (5, 0), or 2 and 5 on x-axis	A1 (2)
		[9]
(a)	B1 for correctly taking out a factor of x M1 for an attempt to factorize their 3TQ e.g. $(x+p)(x+q)$ where $\|pq\| = 9$. So $(x-3)(x+3)$ will score M1 but A0 A1 for a fully correct factorized expression - accept $x(x-3)^2$ If they "solve" use ISW	
S.C.	If the only correct linear factor is (x - 3), perhaps from factor theorem, award B0M1A0 Do not award marks for factorising in part (b)	
	<u>**For the graphs**</u> "Sharp points" will lose the 1st B1 in (b) but otherwise be generous on shape Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.	
(b)	2nd B1 for a curve that starts or terminates at (0, 0) score B0 4th B1ft for a curve that touches (not crossing or terminating) at (a, 0) where their $y = x(x-a)^2$	
(c)	M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only **Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)**	

Q4

Question	Scheme	Marks	AOs
(a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x) \Rightarrow k^2+1x=-2x^2+5x \Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Notes
(a) **M1:** For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification) **A1:** Correct shape and position for both branches. It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour **B1:** Asymptote given as $y=1$. This could appear on the diagram or within the text. Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate. **(b)** **M1:** Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x **A1*:** Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips. Condone if the order of the terms are different $2x^2+k^2-4x=0$ **(c)** **M1:** Deduces that $b^2-4ac=0$ or equivalent for **the given equation.** If a, b and c are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$ Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0 \Rightarrow (x-1)^2=1-\frac{1}{2}k^2 \Rightarrow "1-\frac{1}{2}k^2"=0$ **A1:** $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct **A1:** $k=\pm\sqrt{2}$ and following correct a, b and c if stated

A solution via differentiation would be awarded as follows **M1**: Sets the gradient of the curve $=-2 \Rightarrow -\frac{k^2}{x^2}=-2 \Rightarrow x=(\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2-4x+k^2=0$ **A1**: $2k^2=(\pm)2\sqrt{2}k$ oe **A1:** $k=\pm\sqrt{2}$

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

The point P (1, a) lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) Sketch the curves with the following equations:

(i) $y = (x + 1)^2\ (2 - x)$,

(ii) $y = \dfrac{2}{x}$

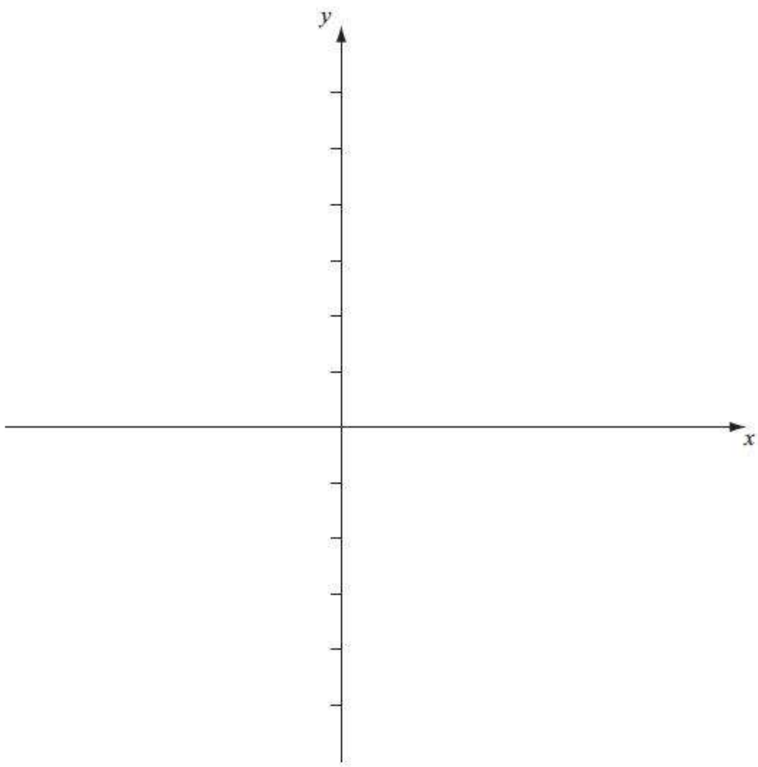
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2\left(2-x\right)=\frac{2}{x}$$

(1)



(Total for Question 1 is 7 marks)

Q2

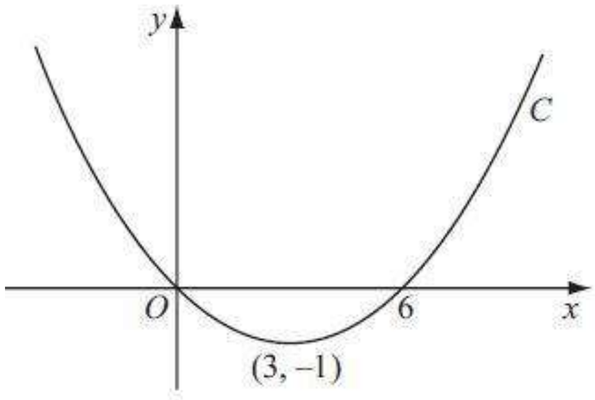


Figure 1

Figure 1 shows a sketch of the curve C with equation y = f (x)

The curve C passes through the origin and through (6, 0)

The curve C has a minimum at the point (3, −1).

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$,

(3)

(b) $y = -\mathrm{f}(x)$,

(3)

(c) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$.

(4)

(Total for Question 1 is 10 marks)

Q3

(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^3 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates (–3, 0) lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

(Total for Question 3 is 7 marks)

Q4

(a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \dfrac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \dfrac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 2$

(3)

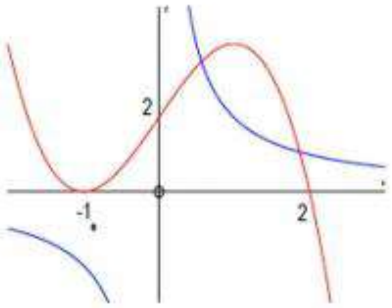
(c) Hence find the range of possible values for c

(4)

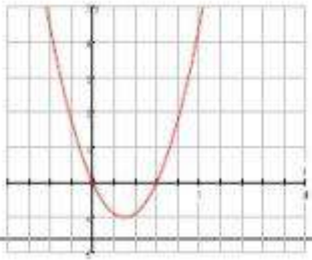
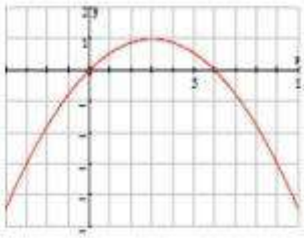
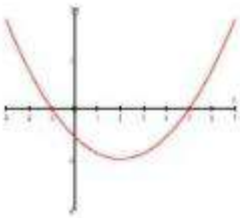
(Total for Question 4 is 11 marks)

Gold Mark Scheme

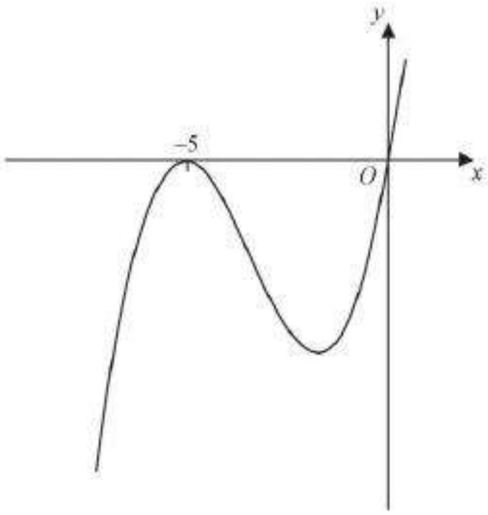
Q1

Question Number	Scheme	Marks	
(a)	$(a=)\ (1+1)^2(2-1)=\underline{4}$ $\quad$ $(1, 4)$ or $y=4$ is also acceptable	B1	(1)
(b)	 (i) Shape or anywhere	B1	
	Min at $(-1, 0)$ … can be -1 on x-axis. Allow $(0, -1)$ if marked on the x-axis. Marked in the correct place, but 1, is B0.	B1	
	$(2, 0)$ and $(0, 2)$ can be 2 on axes	B1	
	(ii) Top branch in 1st quadrant with 2 intersections	B1	
	Bottom branch in 3rd quadrant (ignore any intersections)	B1	(5)
(c)	(2 intersections therefore) $\underline{2}$ (roots)	B1ft	(1) [7]
(b)	1st B1 for shape or Can be anywhere, but there must be one max. and one min. and no further max. and min. turning points. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for minimum at $(-1, 0)$ (even if there is an additional minimum point shown) 3rd B1 for the sketch meeting axes at (2, 0) and (0, 2). They can simply mark 2 on the axes. The marks for minimum and intersections are dependent upon having a sketch. Answers on the diagram for min. and intersections take precedence over answers seen elsewhere. 4th B1 for the branch fully within 1st quadrant having 2 intersections with (not just 'touching') the other curve. The curve can 'touch' the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes, and when the curve looks like two straight lines with a small curve at the join. Allow, for example, shapes like these: 5th B1 for a branch fully in the 3rd quadrant (ignore any intersections with the other curve for this branch). The curve can 'touch' the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes.		
(c)	B1ft for a statement about the number of roots - compatible with their sketch. No sketch is B0. The answer 2 <u>incompatible with the sketch</u> is B0 (ignore any algebra seen). If the sketch shows the 2 correct intersections <u>and</u>, for example, one other intersection, the answer here should be 3, not 2, to score the mark.		

Q2

Question Number	Scheme		Marks
(a)		Shape $\cup$ through (0, 0) (3, 0) (1.5, −1)	B1 B1 B1 (3)
(b)		Shape $\cap$ (0, 0) and (6, 0) (3, 1)	B1 B1 B1 (3)
(c)		Shape $\cup$, <u>not</u> through (0, 0) Minimum in 4^{th} quadrant $(-p, 0)$ and $(6-p, 0)$ $(3-p, -1)$	M1 A1 B1 B1 (4) **10**
	<u>Notes</u>		
	(a) B1: U shaped parabola through origin B1: (3,0) stated or 3 labelled on x axis B1: (1.5, -1) or equivalent e.g. (3/2, -1) (b) B1: Cap shaped parabola in any position B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis B1: (3,1) shown (c) M1: U shaped parabola not through origin A1: Minimum in 4^{th} quadrant (depends on M mark having been given) B1: Coordinates stated or shown on x axis B1: Coordinates stated Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)		

Q3

Question	Scheme	Marks	AOs
(a)	$x^3 + 10x^2 + 25x = x(x^2 + 10x + 25)$	M1	1.1b
	$= x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes

(a) M1: Takes out factor x

A1: Correct factorisation - allow $x(x+5)(x+5)$

(b) M1: Correct shape

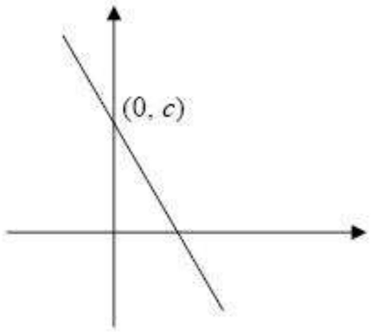
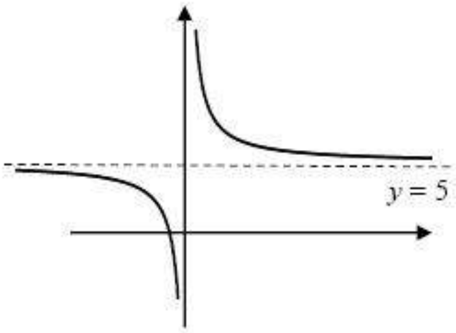
A1ft: Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

(c) M1: May be implied by one of the correct answers for a or by a statement

A1ft: ft from their cubic as long as it meets the x-axis only twice.

A1ft : ft from their cubic as long as it meets the x-axis only twice.

Q4

Question Number	Scheme		Marks
(a)(i)		B1: Straight line with negative gradient anywhere even with no axes.	B1
		B1: Straight line with an intercept at $(0, c)$ or just c marked on the positive y-axis provided the line passes through the positive y-axis. Allow $(c, 0)$ as long as it is marked in the correct place. Allow $(0, c)$ in the body of the script but in any ambiguity, the sketch has precedence. **Ignore any intercepts with the x-axis.**	B1
(a)(ii)		**<u>Either:</u>** For the shape of a $y=\frac{1}{x}$ curve in any position. It must have two branches and be asymptotic horizontally and vertically with no obvious "overlap" with the asymptotes, but otherwise be generous. The curve may bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both vertically and horizontally and the branches must approach the same asymptote **<u>Or</u>** the **equation** $y = 5$ seen independently i.e. whether the sketch has an asymptote here or not. Do not allow $y \neq 5$ or $x = 5$.	B1
		B1: Fully correct graph **and** with a horizontal asymptote on the positive y-axis. The asymptote does not have to be drawn but the **equation** $y = 5$ must be seen. The shape needs to be reasonably accurate with the "ends" not bending away significantly from the asymptotes and the branches must approach the same asymptote. Ignore $x = 0$ given as an asymptote.	B1
	Allow sketches to be on the same axes.		
			(4)

(b)	$\frac{1}{x}+5=-3x+c \Rightarrow 1+5x=-3x^2+cx$ $\Rightarrow 3x^2+5x-cx+1=0$	Sets $\frac{1}{x}+5=-3x+c$, attempts to multiply by x and collects terms (to one side). Allow e.g. ">" or "<" for "=". At least 3 of the terms must be multiplied by x, e.g. allow one slip. The ' = 0' may be implied by subsequent work and provided correct work follows, full marks are still possible in (b).	M1
	$b^2-4ac=(5-c)^2-4\times1\times3$	Attempts to use b^2-4ac with **their** a, b and c from **their equation** where $a=\pm3$, $b=\pm5\pm c$ and $c=\pm1$. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of b^2-4ac. There must be no x's.	M1
	$(5-c)^2>12$*	Completes proof with **no errors or incorrect statements** and with the ">" appearing correctly before the final answer, which could be from $b^2-4ac>0$. Note that the statement $3x^2+5x-cx+1>0$ or starting with e.g. $\frac{1}{x}+5>-3x+c$ would be an error.	A1*
	Note: A minimum for (b) could be, $\frac{1}{x}+5=-3x+c \Rightarrow 3x^2+5x-cx+1(=0)$ (M1) $b^2>4ac \Rightarrow (5-c)^2>12$ (M1A1) If $b^2>4ac$ is not seen then $4\times3\times1$ needs to be seen explicitly.		
			(3)

(c)	$(5-c)^2 = 12 \Rightarrow (c=)5 \pm \sqrt{12}$ or $(5-c)^2 = 12 \Rightarrow c^2 - 10c + 13 = 0$ $\Rightarrow (c=)\frac{-^-10 \pm \sqrt{(-10)^2 - 4\times 13}}{2}$	M1: Attempts to find at least one critical value using the result in (b) or by expanding and solving a 3TQ (See General Principles) (the "= 0" may be implied) A1: Correct critical values in any form. Note that $\sqrt{12}$ may be seen as $2\sqrt{3}$.	M1A1
	$c < "5-\sqrt{12}",\ c > "5+\sqrt{12}"$	Chooses outside region. The '0 <' can be ignored for **this** mark. So look for $c <$ their $5-\sqrt{12}$, $c >$ their $5+\sqrt{12}$. This could be scored from $5+\sqrt{12} < c < 5-\sqrt{12}$ or $5-\sqrt{12} > c > 5+\sqrt{12}$. Evidence is to be taken from their answers not from a diagram.	M1
	$0 < c < 5-\sqrt{12},\ c > 5+\sqrt{12}$	Correct ranges including the '0 <' e.g. answer as shown or each region written separately or e.g. $\left(0, 5-\sqrt{12}\right), \left(5+\sqrt{12}, \infty\right)$. The critical values may be un-simplified but must be at least $\frac{10+\sqrt{48}}{2}, \frac{10-\sqrt{48}}{2}$. Note that $0 < c < 5-\sqrt{12}$ **and** $c > 5+\sqrt{12}$ would score M1A0.	A1
	Allow the use of x rather than c in (c) but the final answer must be in terms of c.		
			(4)
			(11 marks)